SOLUTIONS

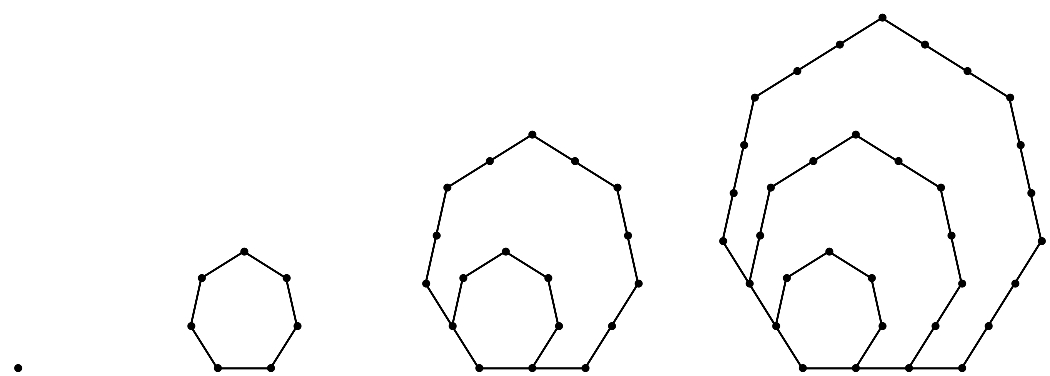
No problem is ever permanently closed. Any comments, new solutions, or new insights on old problems are always welcomed by the problem editor.

67. [1994, 47; 1995, 43] *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Show that one more than four times the product of two consecutive even or odd numbered triangular numbers is a square.

Comment by Joseph B. Dence, University of Missouri-St. Louis, St. Louis, Missouri.

Let $\{P(2n+1,k)\}_{k=1}^{\infty}$ be the polygonal numbers of order $2n+1$. For example, $\{P(3,k)\}_{k=1}^{\infty}$ are the triangular numbers: $P(3,1) = T_1 = 1$, $P(3,2) = T_2 = 3$, $P(3,3) = T_3 = 6, \ldots$. Similarly, the first few heptagonal numbers are $P(7,1) = 1$, $P(7,2) = 7$, $P(7,3) = 18$, $P(7,4) = 34, \ldots$.



1 7 18 34

By induction, $P(7,k) = k(5k-3)/2$, and in general any polygonal number of odd order is given explicitly by

$$P(2n+1,k) = \frac{k}{2}[(2n-1)k + (3-2n)], \quad \text{for} \quad n \geq 1.$$

After some algebra, we find

$$\begin{aligned} &P(2n+1,k)P(2n+1,k+2) \\ &= \frac{1}{4}\left([k[(2n-1)(k+2)+(3-2n)]]^2 + 2(3-2n)k[(2n-1)(k+2)+(3-2n)]\right), \end{aligned}$$