SEQUENTIAL G_δ-SETS AND MEASURES

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For any set Y we equip the power set $P(Y)$ with a compact Hausdorff topology by taking as subbasic open sets all sets of the forms $\{D \subseteq Y \mid y \in D\}$ and $\{D \subseteq Y \mid y \notin D\}$ where y varies throughout Y. For any class X of sets, the union of this class is a set $\cup X$ and its power set $P(\cup X)$ may be equipped with a topology as above. Considered as a subset of $P(\cup X)$, X might be both sequentially closed in $P(\cup X)$ and a sequential G_δ-set (the intersection of countably many sequentially open sets) in $P(\cup X)$. If this is the case, we say that X is a *Mazur class*.

For any set Y, we say that Y is *Mazur reducible* provided every Mazur class of subsets of Y that contains all finite subsets of Y must also contain Y itself. A cardinal number m is said to be Mazur reducible if there is a Mazur reducible set of cardinality m. (If a set is Mazur reducible, then so are all sets of the same cardinality as this set.)

Mazur [2] proved that all cardinal numbers less than the smallest inaccessible cardinal number are Mazur reducible. Ulam [3] proved that each nonmeasurable cardinal number is non-realmeasurable provided 2^ω is non-realmeasurable.

For any set Y, we say that Y is *modified Mazur reducible* provided every Mazur class of subsets of Y that is finitely additive and contains all finite subsets of Y must also contain Y itself. As above, we say that a cardinal number m is modified Mazur reducible if there is a modified Mazur reducible set of cardinality m.

It follows from [1, Theorem 1] that all Mazur reducible cardinals are modified Mazur reducible.

Theorem. Every nonmeasurable cardinal number is modified Mazur reducible if and only if 2^ω is modified Mazur reducible.

Proof. Since 2^ω is nonmeasurable, it remains to prove that if 2^ω is modified Mazur reducible, then m is modified Mazur reducible for every nonmeasurable cardinal number m.

Lemma 1. Let Y denote a set of nonmeasurable cardinality. If X is a Mazur class of subsets of Y that is finitely additive and contains all finite subsets of Y, then for every