ANOTHER PROOF THAT THE CLOSED UNIT INTERVAL IS A CONTINUOUS IMAGE OF THE CANTOR SET

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It is well known ([1] for example) that the closed unit interval is the continuous image of the Cantor Set. The usual proof proceeds as follows.

The standard Cantor middle thirds set can be realized as the set

$$C = \left\{ \sum_{n=1}^{\infty} \frac{t_n}{3^n} : t_n = 0 \quad \text{or} \quad t_n = 2 \right\}.$$

One may then construct a map $f : C \to I$ by

$$f : \sum_{n=1}^{\infty} \frac{t_n}{3^n} \mapsto \sum_{n=1}^{\infty} \frac{\phi(t_n)}{2^n},$$

where $\phi(0) = 0$ and $\phi(2) = 1$. One then checks that f is continuous and surjective.

This is an elegant proof that cannot be improved upon.

That which follows is an alternate proof of the same result. The main virtue of this approach is that it introduces the reader to some classical results of general topology and to the concept of the code space which is very useful in the growing area of dynamical systems.

1. The Code Space. In the sequel, we will let S denote the set of all functions from N (the set of natural numbers $1, 2, 3, \dots$) to the set $\mathbb{Z}_2$ (the integers $\pmod 2$) which contains two elements denoted by 0 and 1). We will topologize S by describing a basis. For every $x \in S$ let $N(x, 0) = S$. For every $x \in S$ and $n \in \mathbb{N}$, let

$$N(x, n) = \{y \in S : y(i) = x(i), 1 \leq i \leq n\}.$$