STIRLING'S FORMULA: AN APPLICATION OF THE CENTRAL LIMIT THEOREM

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The large factorials are approximated through the use of Stirling's formula:

$$n! \simeq \sqrt{2\pi}\ n^{n+1/2}e^{-n}.$$

The proof of the Stirling's formula can be found in many texts, such as [1], [2], and [3].

In this short note Stirling's formula is derived as an application of the Central Limit Theorem. Thus, this proof can be introduced in a mathematical statistics course.

Let $X_1, X_2, \ldots, X_n$ be a random sample from an exponential distribution with mean 1.

By the Central Limit Theorem the limiting distribution of

$$Z_n = \frac{\sum_{i=1}^n X_i - n}{\sqrt{n}}$$

is standard normal.

That is,

$$Z_n \xrightarrow{d} Z \sim N(0,1) \ \text{ as } \ n \to \infty.$$

Thus, for every x,

$$P(Z_n \le x) \to P(Z \le x) \ \text{ as } \ n \to \infty.$$

Since $X_i \sim \exp(1), i = 1, 2, \ldots, n$, and all independent, $\sum_{i=1}^{n} X_i$ has a Gamma distribution with probability density function

$$f(t) = \frac{t^{n-1}e^{-t}}{(n-1)!} \ \ t \ge 0, \ \text{ and zero otherwise.}$$