# A GENERALIZED EXPONENTIAL FUNCTION

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Let $p$ be a positive integer. Using the ratio test, it can be shown that

$$S(p) = \sum_{n=0}^{\infty} \frac{x^n}{(pn)!} \tag{1}$$

converges for $|x| < \infty$. This follows from the fact

$$\lim_{n\to\infty} \frac{(pn)!}{\big(p(n+1)\big)!} = 0.$$

The purpose of this paper is to express (1) in terms of a hypergeometric function. The special functions that will be used are reviewed first.

The *factorial function* is defined by

$$(a)_n = a(a+1)\cdots(a+n-1)$$

for any $a$ and for $n \geq 1$ and, $(a)_0 = 1$, for $a \neq 0$. In particular, $n! = (1)_n$ and, from page 9 of [1],

$$(a)_{nk} = k^{nk} \prod_{i=1}^{k} \left(\frac{a+i-1}{k}\right)_n .$$