SOLUTIONS

No problem is ever permanently closed. Any comments, new solutions, or new insights on old problems are always welcomed by the editor.

17. *Proposed by Stanley Rabinowitz, Westford, Massachusetts.*

Let $ABCD$ be an isosceles tetrahedron. (An isosceles tetrahedron is a tetrahedron whose opposite edges are equal.) Denote the dihedral angle at edge AB by $\angle AB$. Prove that

$$\frac{AB}{\sin\angle AB}=\frac{AC}{\sin\angle AC}=\frac{AD}{\sin\angle AD}\ .$$

Solution by the proposer.

Let AH be the altitude from A to face BCD and let AE be the altitude from A to BC in face ABC. Then triangle AHE is a right triangle with $\angle AEH=\angle BC$. Hence $\sin\angle BC=AH/AE$. But $AH=3V/K$, where V is the volume of the tetrahedron and K is the area of any face; and $AE=2K/BC$. Therefore $\sin\angle BC=3V(BC)/2K^2$ and hence

$$\frac{\sin\angle BC}{BC}$$

is constant. Since BC was arbitrary, this is true for all six edges of the tetrahedron.

18. *Proposed by Jayanthi Ganapathy, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.*

Let f be differentiable on an interval of the form (M,∞). Suppose

$$\lim_{x\to\infty}\big(f(x)+xf'(x)\big)=\alpha\ ,$$

where α is finite. Prove

$$\lim_{x\to\infty}f(x)\qquad\text{and}\qquad\lim_{x\to\infty}f'(x)$$

exist and evaluate these limits.