CONSECUTIVE COMPOSITE VALUES OF A QUADRATIC POLYNOMIAL

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Let a, b, and c be integers such that $b^2 - 4ac$ is not a perfect square. We are interested in finding sequences of integers n such that $an^2 + bn + c$ is composite. Of course, if $b^2 - 4ac$ is a perfect square, then $an^2 + bn + c$ is always composite. We follow along the lines of Garrison in [1].

Let $\mathcal{P} = \{p_t\}_{t=0}^{+\infty}$ be the sequence of primes such that $p_0 = 2$, $p_t < p_{t+1}$ and for all $p \in \mathcal{P}$ $((b^2 - 4ac)/p) = +1$, where (here and below) (m/p) denotes the Legendre symbol. Then $\mathcal{P}$ contains the prime divisors of all the $an^2 + bn + c$. Let

$$P(t) = \prod_{k=0}^{t} p_k$$

and let

$$C(t) = \{n : (an^2 + bn + c, P(t)) > 1\} .$$

For $i = 1$ and 2 let a_{ik} be the solutions to $an^2 + bn + c \equiv 0 \pmod{p_k}$ and let

$$S(t) = \{x : x \not\equiv 1 \pmod 2 \ \text{ and } \ x \not\equiv a_{ik} \pmod{p_n}, h = 1, \ldots, t\} .$$