

SOLUTIONS

No problem is ever permanently closed. Any comments, new solutions, or new insights on old problems are always welcomed by the editor.

6. *Proposed by Curtis Cooper and Robert E. Kennedy, Central Missouri State University, Warrensburg, Missouri.*

Prove

$$\sum_{n \leq x} \frac{1}{3n-2} = \frac{1}{3} \log(3x-2) + \frac{1}{6} \log 3 + \frac{\pi}{6\sqrt{3}} + \frac{\gamma}{3} + O\left(\frac{1}{x}\right),$$

where \log is the natural log and γ is Euler's constant.

Comment by Don Redmond, Southern Illinois University at Carbondale, Carbondale, Illinois.

The problem as given is to find an asymptotic expansion for

$$\sum_{n \leq x} \frac{1}{3n-2}.$$

If we look in Ramanujan's notebooks (B.C. Berndt, Ramanujan's Notebooks, part I, p. 185) we find the following in Chapter 8, Entry 7. If x is a positive integer and a and b are arbitrary complex numbers, then

$$\Psi\left(\frac{a}{b} + x + 1\right) - \Psi\left(\frac{a}{b} + 1\right) = b \sum_{k=1}^x \frac{1}{a + bk}.$$

Here

$$\Psi(z) = \frac{\Gamma'}{\Gamma}(z).$$

In Abramowitz and Stegun (Handbook of Mathematical Functions, p. 259) we find an asymptotic expansion for $\Psi(z)$ so that we may