A PUZZLE IN KELLEY'S APPENDIX

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In the appendix of John L. Kelley's General Topology there is a system of axiomatic set theory consisting of eight axioms and an axiom scheme from the unpublished lecture notes of Anthony P. Morse. Its presence there poses a puzzle: What is the relation of axiomatic set theories to general topology? Is it the same as that of Cantorian set theory? If not, what are the differences and their consequences for general topology? The answers to these questions will be given here after this sytem has been compared to the two standard systems of axiomatic set theory, Zermelo-Fraenkel (abbreviated ZF) and von Neumann-Bernays-Gödel (abbreviated VBG) and the metamathematics of these theories has been discussed in detail.

These set theories are first order formal theories. A formal theory consists of three parts, specific rules for the formation of its language in the setting of symbolic logic, rules for the formation of sentences (formulas) within this language, and rules for the formation of theorems as deduced from its axioms. A first order theory deals with one kind of object only. In these theories that object is either a set or class. These theories are embedded in first order logic which gives them an artificial aspect since one formal theory is then developed within another. This artificialness has great consequences for these theories.

These axiomatic set theories differ from Cantorian set theory in another important respect. Only the relationships among sets or classes are investigated. Thus, they are structural theories in which elements that are not sets or classes are ignored. Class, as used here, means a collection that is not necessarily a set.