HOW MATHEMATICS GROWS

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Mathematics grows on all levels. One doesn't need a Ph.D. — as is the case in other sciences — to make interesting and significant contributions to the field. Even at the lower levels, students can be challenged to initiate research projects, and to develop novel solutions to problems of their own devising.

In this paper, we explore some of the procedures by which mathematics grows. We also offer several original examples of varying difficulty that demonstrate these procedures. Our hope is that instructors can use these examples, and the methodology that underlies them, to motivate students to think of mathematics as an enterprise of which they (the students) are a part. If students (and instructors) feel that there are still many problems on the lowest levels of mathematics in need of formulation and solution, courses in mathematics can become more stimulating and alive.

1. Observation and Explanation

Typically, the starting point for the mathematician is observation. After observing some unusual — or not so unusual — event, he/she will seek to explain what he/she has observed mathematically.

As an example, suppose our mathematician visits a Las Vegas Casino, and observes the following four-card magic trick: the dealer presents a brand new fifty-two card deck and asks the audience to cut the cards as many times as they please, once at a time (here, cutting once at a time means cutting the top of the deck, putting the cut cards to the right, and placing the bottom cards on top of them). The dealer then deals four cards to thirteen different people, and "magically" each person receives four cards of the same kind — i.e., four aces, four kings, four deuces, etc.

His curiosity piqued, our mathematician might naturally seek to explain this "phenomenon". He can do so in a non-technical and understandable manner using a standard technique of geometric