No problem is ever permanently closed. Any comments, new solutions, or new insights on old problems are always welcomed by the problem editor.

161. [2006, 147] *Proposed by José Luis Díaz-Barrero, Universidad Politècnica de Cataluña, Barcelona, Spain.*

Show that

$$\int_0^1 \sqrt[3]{1+\ln(1+x)}\,dx \int_0^1 \sqrt[3]{(1+\ln(1+x))^2}\,dx < 2\ln 2.$$

Solution by Joe Howard, Portales, New Mexico. We use an inequality due to Chebyshev found on page 135 (Problem 75) of G. Klambauer, *Problems and Propositions in Analysis*, (1979), Marcel Dekker. With $p(x) = 1$ and f and g monotonically increasing on $[0,1]$, we have

$$\int_0^1 f(x)dx \cdot \int_0^1 g(x)dx \le \int_0^1 f(x)\cdot g(x)dx.$$

Assuming

$$f(x) = \big(1+\ln(1+x)\big)^{\frac{1}{3}} \qquad (f'(x) > 0 \text{ on } [0,1])$$

and

$$g(x) = \big(1+\ln(1+x)\big)^{\frac{2}{3}} \qquad (g'(x) > 0 \text{ on } [0,1])$$

the conditions of the inequality are met. Now

$$\int_0^1 \big(1+\ln(1+x)\big)dx = 1 + \int_1^2 \ln t dt = 1 + \big[t\ln t - t\big]_1^2 = 2\ln 2.$$

Also, from the proof of Problem 75 (above), f or g must be constant to have equality. Hence, the inequality is strict.

Also solved by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, Missouri (jointly); Kenneth B. Davenport,