No problem is ever permanently closed. Any comments, new solutions, or new insights on old problems are always welcomed by the problem editor.

165. [2007; 151] *Proposed by José Luis Díaz-Barrero, Universidad Politècnica de Cataluña, Barcelona, Spain.*

Let n be a positive integer. Prove that

$$\frac{1}{2n}\left(\sum_{k=1}^{n}\sqrt{1+\left(F_k\binom{n}{k}\right)^2}\right)^2 \geq F_{2n},$$

where F_n is the nth Fibonacci number defined by $F_0 = 0$, $F_1 = 1$ and for all $n \geq 2$, $F_n = F_{n-1} + F_{n-2}$.

Solution I by Brian Bradie, Christopher Newport University, Newport News, Virginia. For $n = 1$, we have

$$\frac{1}{2}\left(\sqrt{1+F_1^2}\right)^2 = \frac{1}{2}\left(\sqrt{1+1}\right)^2 = 1 = F_2;$$

whereas, for $n = 2$, we have

$$\begin{aligned}\frac{1}{4}\left(\sqrt{1+(2F_1)^2}+\sqrt{1+F_2^2}\right)^2 &= \frac{1}{4}\left(\sqrt{1+4}+\sqrt{1+1}\right)^2 \\ &= \frac{1}{4}\left(7+2\sqrt{10}\right) > 3 = F_4.\end{aligned}$$

To proceed further, first note that by a straightforward application of mathematical induction we can show that $F_m > m$ for all $m \geq 6$. Now, let $n \geq 3$. Then

$$\frac{1}{2n}\left(\sum_{k=1}^{n}\sqrt{1+\left(F_k\binom{n}{k}\right)^2}\right)^2 > \frac{1}{2n}\left(\sum_{k=1}^{n}F_k\binom{n}{k}\right)^2 = \frac{1}{2n}F_{2n}^2,$$