No problem is ever permanently closed. Any comments, new solutions, or new insights on old problems are always welcomed by the problem editor.

169. *Proposed by Dorin Marghidanu, Colegiul National "A. I. Cuza", Corabia, Romania.*

Let $0 < a, b, c < 1$. Prove that

$$2^a(b+c)^{1-a} + 2^b(c+a)^{1-b} + 2^c(a+b)^{1-c} < 4(a+b+c).$$

Solution by Tuan Le (student), Fairmont High School, Anaheim, California. Since $0 < a < 1$, applying Bernoulli's inequality, we have

$$\left(\frac{2}{b+c}\right)^a = \left(1 + \frac{1-\frac{b+c}{2}}{\frac{b+c}{2}}\right)^a \leq 1 + \frac{a(1-\frac{b+c}{2})}{\frac{b+c}{2}}.$$

Multiplying both sides of this inequality by $b+c$, we obtain

$$2^a(b+c)^{1-a} \leq 2a + b + c - a(b+c).$$

Similarly, we also obtain

$$2^b(a+c)^{1-b} \leq 2b + a + c - b(a+c)$$

$$2^c(a+b)^{1-c} \leq 2c + a + b - c(a+b).$$

Adding these inequalities together and again using the fact that $0 < a, b, c < 1$, we obtain

$$2^a(b+c)^{1-a} + 2^b(a+c)^{1-b} + 2^c(a+b)^{1-c}$$

$$\leq 4(a+b+c) - 2(ab+bc+ac) < 4(a+b+c).$$