

**ON THE SOLUTION OF THE CUBIC PYTHAGOREAN  
DIOPHANTINE EQUATION  $x^3 + y^3 + z^3 = a^3$**

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**1. Introduction.** For hundreds of years, mathematicians, including Euler, Vieta, Binet, Ramanujan and others have been fascinated and perplexed by the 3.1.3 cubic Pythagorean Diophantine equation,  $x^3 + y^3 + z^3 = a^3$ . Though complex partial solutions have been provided by these renowned mathematicians, a complete solution of this equation in integers still eludes us. In this paper, we propose to provide two independent relatively simple solutions of this intriguing equation. Though a complete general solution in integers is still not at hand, the solutions derived for the special cases described here do provide significant insight into this most fascinating equation and prove unequivocally that we can easily produce an infinite number of solutions in the family of equations satisfying the relationship  $x^3 + y^3 + z^3 = a^3$ .

**2. First Solution.** Consider the following subset of known solutions to the 3.1.3 cubic Pythagorean Diophantine equation,  $x^3 + y^3 + z^3 = a^3$ , in order to determine certain facts and relationships.

$$3^3 + 4^3 + 5^3 = 6^3$$

$$4^3 + 17^3 + 22^3 = 25^3$$

$$16^3 + 23^3 + 41^3 = 44^3$$

$$16^3 + 47^3 + 108^3 = 111^3$$

$$64^3 + 107^3 + 405^3 = 408^3$$

$$64^3 + 155^3 + 664^3 = 667^3.$$

Perhaps the most crucial insight of all is to initially rearrange the first equation in the above series to read  $4^3 + 5^3 + 3^3 = 6^3$ . This is a critical first step for two reasons. First, there are now two equations for each  $x$  value, and  $x$  is always a power of 4,

$$\text{i.e., } x = (4^n)^3 \quad \text{where } n = 1, 2, 3, \dots \quad (1)$$