COMPLEMENTARY INTEGER SEQUENCES THAT HAVE ONLY INITIAL COMMON MOMENTS

Hang Chen

The k^{th} *moment*, $m_k(R)$, of a nonnegative integer sequence $R = \{r_i\}_1^n$ of length n is defined to be the sum of the k^{th} powers of the elements, that is,

$$m_k(R) = \sum_{i=1}^{n} r_i^k.$$

It is convenient to assume that the 0^{th} power of any number is 1. Two equal-length sequences R and Q of nonnegative integers are said to share the k^{th} moment if $m_k(R) = m_k(Q)$. The *common moment set* of R and Q is $P = \{k \mid m_k(R) = m_k(Q)\}$. The *initial interval* of the common moment set is defined to be $P_0 = \{0, 1, 2, \ldots, m(R,Q)\}$, where $m(R,Q) = \max\{j \mid m_k(R) = m_k(Q), 0 \leq k \leq j\}$. Therefore, the common moment set is $P = P_0 \cup A$, where $A \subset \{m(R,Q)+2, m(R,Q)+3, \ldots\}$. If R and Q are identical sequences, we interpret $p = \infty$. If R and Q are two distinct sequences, the common moment set P is a finite set. We shall discuss nonidentical sequences in this paper.

Chen, Erdős, and Schwenk [2] studied the comment moment sets for the score sequences of complementary tournaments and showed that such a common moment set is $P = \{0, 1, 2, \ldots, 2p\} \cup A$, where $p \geq 0$ and $A \subset \{2p+3, 2p+4, \ldots\}$. Chen [1] provided parallel results for degree sequences of complementary graphs.

Two nonnegative integer sequences $R = \{r_i\}_1^n$ and $Q = \{q_i\}_1^n$ are said to be *complementary* if $r_i + q_i$ is a constant for $i = 1, 2, \ldots, n$. In this paper, we show that the initial interval of the common moment set for complementary sequences is $P = \{0, 1, 2, \ldots, 2p\}$, that is, $m(R,Q) = 2p$ for some $p \geq 0$. We present complementary integer sequences that share only the initial moments. For any given integer $p \geq 0$, we shall construct complementary integer sequences of length 4^p that have the common moment set $P = \{0, 1, 2, \ldots, 2p\}$.

For two sequences of length n both arranged in nonincreasing order, we say that $R = \{r_i\}_1^n$ *dominates* $Q = \{q_i\}_1^n$ if there is an index i_0 such that $r_{i_0} > q_{i_0}$ and $r_i = q_i$ for $i < i_0$. For example, $R = \{6, 5, 3\}$ dominates $Q = \{6, 4, 4\}$. For distinct sequences, one must always dominate the other.