SOLUTIONS

No problem is ever permanently closed. Any comments, new solutions, or new insights on old problems are always welcomed by the problem editor.

141. [2003, 200] *Proposed by Kenneth B. Davenport, Frackville, Pennsylvania.*

(a) Show that

$$\sum_{k=1}^{n-1} \frac{1}{1-\tan^2\left(\frac{k\pi}{2n}\right)} = \frac{n-1}{2},$$

for $n = 3, 5, 7, 9, \ldots$.

(b) Show that

$$\sum_{k=1}^{n-1} \frac{1}{1+\tan^2\left(\frac{k\pi}{2n}\right)} = \frac{n-1}{2},$$

for $n = 2, 3, 4, 5, \ldots$.

Solution by Joe Howard, Portales, New Mexico. Note that $\cos\theta = -\cos(\pi-\theta)$ and $\sec\theta = -\sec(\pi-\theta)$. By pairing in this way (Ex: $\cos\frac{\pi}{3} + \cos\frac{2\pi}{3} = 0$) it follows that

$$\sum_{k=1}^{n-1} \cos\frac{k\pi}{n} = 0 \text{ and } \sum_{k=1}^{n-1} \sec\frac{k\pi}{n} = 0 \text{ for } n = 3, 5, 7, 9, \ldots .$$

Also, $\cos\frac{\pi}{2} = 0$ so

$$\sum_{k=1}^{n-1} \cos\frac{k\pi}{n} = 0 \text{ for } n = 2, 4, 6, 8, \ldots .$$