AN INVESTIGATION OF THE SET OF ANS NUMBERS

Norman E. Elliott and Dan Richner

The investigation of properties of the integers can lead to fascinating and challenging questions. This is what makes the branch of mathematics known as "number theory" the so-called "playground of mathematicians." In fact, some of the simplest sounding and accessible ideas concerning the integers can result in such questions. Here, we will introduce and discuss one such idea. All undefined terminology and notation is that of [2] and its later editions.

Definition. A positive integer is called an *ans number* if it can be expressed as the difference of squares of two prime integers.

Thus, 40 and 45 are ans numbers since

$$40 = 7^2 - 3^2 \text{ and } 45 = 7^2 - 2^2.$$

On the other hand, 44 is not ans since it cannot be expressed as the difference of squares of two primes. A procedure to determine whether or not a particular positive integer is ans can be developed.

Noting that in order to show that a positive integer, n, is ans, we must find two primes p, q, such that

$$p + q = a, \quad p - q = b, \quad \text{and} \quad n = ab.$$

Hence, we see that

$$p = \frac{(a+b)}{2} \qquad \text{and} \qquad q = \frac{(a-b)}{2}.$$

Therefore, we can state the following theorem.

Theorem 1. The positive integer n is ans if and only if there exist two integers a, b such that

$$n = ab$$

and

$$\frac{(a+b)}{2} \qquad \text{and} \qquad \frac{(a-b)}{2}$$