SOLUTIONS

No problem is ever permanently closed. Any comments, new solutions, or new insights on old problems are always welcomed by the problem editor.

129. [1999, 196] *Proposed by Kenneth B. Davenport, 301 Morea Road, Box 491, Frackville, Pennsylvania.*

Let $k \geq 0$ and $i \geq 1$ be integers. Prove that

$$\sum_j \left\langle {k \atop j} \right\rangle \binom{k+i-j}{k+1} = \sum_{m=1}^{i} m^k,$$

where

$$\left\langle {k \atop j} \right\rangle$$

denotes an Eulerian number.

Solution by the proposer and Carl Libis, Antioch College, Yellow Springs, Ohio. Here, an Eulerian number

$$\left\langle {k \atop j} \right\rangle$$

is the number of permutations $\pi_1\pi_2\cdots\pi_k$ of $\{1, 2, \ldots, k\}$ that have j ascents, namely, j places where $\pi_i < \pi_{i+1}$. To prove this result we need Worpitsky's identity from R. L. Graham, D. E. Knuth, and O. Patashnik, *Concrete Mathematics,* 2nd ed., Addison-Wesley Publishing Company, Reading, Massachusetts, 1994, p. 269, i.e.

$$m^k = \sum_j \left\langle {k \atop j} \right\rangle \binom{m+j}{k}$$