AN ALGORITHMIC VERSION OF KUHN'S LONE-DIVIDER METHOD OF FAIR DIVISION

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Among the various types of methods for accomplishing a fair division are those commonly called "lone-divider" (we shall not consider other types of methods here). The first such method, for $n = 3$ participants, is credited to Hugo Steinhaus [4]. In 1967, Harold Kuhn [5] became the first to publish a lone-divider method for arbitrarily many participants. Although Kuhn's method has become popular and is discussed in the recent texts [1] and [6], and is even mentioned in lower-level undergraduate texts such as [7], it is rarely if ever restated; this is likely due to the facts that (1) Kuhn's statement of the method is at least partially existential, rather than algorithmic, and (2) Kuhn's proof relies on the Frobenius-König theorem. It is the purpose of this note to introduce an algorithmic version of Kuhn's procedure, along with proof of uniqueness of a set for which Kuhn proves existence only.

We begin by stating the method. Strategies of the various individuals involved will be discussed afterward, followed by proofs of claims made in the statement of the method. We make the common assumption that each player's preferences are determined by a finitely additive, nonatomic measure. All other undefined terms are as in [1].

Lone-Divider Algorithm for n Participants.

Step 0. The n individuals are ordered in a random fashion into rank 1, rank 2, …, rank n. The lower the number, the higher the rank. Rank 1 is highest. The individual in rank n is the *divider;* the others are *choosers.* The identity of the divider is revealed, but the ranks of the choosers are not revealed until after Step 2.

Step 1. The divider cuts the object into n pieces.

Step 2. The choosers mark acceptable pieces; each chooser must mark at least one piece. This must be done in such a manner that no chooser is aware of another chooser's marks when making their marks.

Step 3.

<u>Definition</u>. A set of choosers is called *decidable* if it contains no subset of k choosers who marked a total of $k - 1$ or fewer different acceptable pieces between them.

(i). If the set of all choosers is decidable, execute the "decidable allocation procedure."