THE TOWER OF HANOI PROBLEM AND MATHEMATICAL THINKING

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Too often students leave a course in mathematics with the impression that mathematics can only be done in a statement-proof style, and that only especially gifted people are capable of originating such work. They fail to appreciate that conjecturing is an integral part of the mathematical process, and that this does not necessarily require extraordinary talent. How can this false impression be altered? One way is by allowing students to see that they themselves are able to make conjectures on a substantial mathematical problem. Initial conjectures can be modified or discarded if further checking shows them to be incorrect. Verification of the conjecture involves its proof. By carrying out, either individually or in a group, both parts of the mathematical process-conjecture and proof, students gain a far better understanding of mathematical thinking. However, conjecturing is in itself a valuable exercise, even if it is not combined with proving the conjecture.

In this paper the solution of a problem related to the Tower of Hanoi problem is given. The proof only uses mathematical induction, so it is within the reach of many students. In the Tower of Hanoi problem there are three poles and some rings, no two the same size. Initially, the rings are placed on a single pole A. Neither initially nor during play is a larger ring ever above a smaller ring. A move consists of taking the top ring from one pole and placing it on another pole. The Tower of Hanoi problem is then to find the least number of moves required to move all rings from the "initial pole" A to the "terminal pole" C. The other pole B will be called the "intermediate pole."

A *position* will mean a permitted arrangement of the rings on the poles *with the poles labeled.* The problem is the following: **how can one tell whether a particular position actually occurs during the transfer of rings that takes place in the Tower of Hanoi problem?** At this point the reader might try to find the correct conjecture himself. The only approach appears to be the empirical one, that is, examining the actual positions which occur. It is interesting to ask how many cases students examine on the average before the correct conjecture is found.

Two numbers have the same parity if they are either both even or both odd, otherwise they have opposite parity. To state the conjecture concisely, let's agree that there are n rings and that they are numbered in increasing size from 1 to n, n being the largest ring. Here then is the correct conjecture.

A position is attained during the transfer process if and only if the following two conditions are satisfied: