

SOLUTIONS

No problem is ever permanently closed. Any comments, new solutions, or new insights on old problems are always welcomed by the problem editor.

104. [1997, 35; 1998, 59–61] *Proposed by Kenneth Davenport, Box 491, Frackville, Pennsylvania.*

Show that

$$1 \cdot \sin \frac{\pi}{2n} + 3 \cdot \sin \frac{3\pi}{2n} + 5 \cdot \sin \frac{5\pi}{2n} + \cdots + (2n-1) \sin \frac{(2n-1)\pi}{2n} = n \csc \frac{\pi}{2n}.$$

Solution III by Paul S. Bruckman, Edmonds, Washington.

Let

$$o_k = \frac{1 - (-1)^k}{2}.$$

Then,

$$\sum_{k=0}^{2n-1} k \sin \frac{k\pi}{2n} \cdot o_k = n \cdot \csc \frac{\pi}{2n}, \quad n = 1, 2, \dots$$

is easily evaluated using complex variables. That is,

$$\sin x = \frac{1}{2i}(e^{ix} - e^{-ix}),$$