SOLUTIONS

No problem is ever permanently closed. Any comments, new solutions, or new insights on old problems are always welcomed by the problem editor.

101. [1997, 34] *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Find the coefficient of x^{1997} (in closed form) in the expansion of

$$\sqrt{2x^2 - 3x^3}.$$

Solution by Joseph Wiener, University of Texas-Pan American, Edinburg, Texas.

Write the given radical in the form $\sqrt{2}|x|(1-3x/2)^{1/2}$ and find the coefficient k_{1996} of x^{1996} in the binomial expansion of $(1-3x/2)^{1/2}$, that is,

$$k_{1996} = \binom{1/2}{1996}\left(\frac{3}{2}\right)^{1996},$$

where

$$\binom{1/2}{1996} = \frac{(1/2)!}{1996!(1/2-1996)!} = \frac{\Gamma(3/2)}{1996!\Gamma(-1996+1/2)}.$$

It remains to calculate the values of the gamma function,

$$\Gamma(3/2) = \frac{1}{2}\sqrt{\pi}, \qquad \Gamma\left(-1996+\frac{1}{2}\right) = \frac{4^{1996}(1996)!}{(2\cdot 1996)!}\sqrt{\pi}.$$

Then

$$k_{1996} = \frac{1}{2}\cdot\frac{3992!}{4^{1996}\cdot(1996!)^2}$$