PARABOLAS IN TAXICAB GEOMETRY

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1. Introduction. Reynolds [1] raised some open questions concerning the definition of taxicab parabolas. Moser, Kramer [2], and Iny [3] looked into these questions but did not answer all of them. We would like to provide an analysis to the solutions with more details in this paper.

Let's first take a look at the definition of taxicab metric.

Definition 1.1. Let $d_T : R^2 \times R^2 \to [0, \infty)$
be defined as

$$d_T((a_1, a_2), (b_1, b_2)) = |a_1 - b_1| + |a_2 - b_2|.$$

Then (d_T, R^2) forms a *metric space* and d_T is called the *taxicab metric* on R^2.

In R^2, there are several different but equivalent ways to define straight lines using Euclidean metric. However, Chen [4] showed that these ways are no longer equivalent when Euclidean metric is replaced by taxicab metric. Liu [5] used the idea of a line being the bisector of two points to define the taxicab bisector (or taxicab line).

Definition 1.2 Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be two points in R^2. Then the set

$$D = \{(x, y) | |x - x_1| + |y - y_1| = |x - x_2| + |y - y_2|\}$$

is called the *taxicab bisector* (or the *taxicab line*) of the points P_1 and P_2.

Since a parabola in Euclidean geometry is the set of the points which are equidistant to a fixed point; called the focus, and a fixed line; called the directrix, we need to discuss the distance between a point and a taxicab bisector before defining the taxicab parabola.

Theorem 1.1.[1] The shortest taxicab distance from a point to a Euclidean line is either the horizontal distance or the vertical distance whichever is smaller.

Liu [5] showed that there are three types of taxicab bisectors determined by the relation between the difference of x-coordinates and that of y-coordinates of P_1 and P_2. Two of them are Euclidean line or the union of a Euclidean line segment and