MIXED INSURANCE RISK MODELS

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1. Introduction. Traditionally, the distribution of aggregate claims of a portfolio has been a central topic in risk theory. The discussion has focused on two problems: the choice of the distribution and its numerical evaluation by means of an approximation. Consider the collective risk model where the aggregate claim random variable S for a portfolio of insurance policies over a fixed period can be expressed as

$$S = X_1 + X_2 + \cdots + X_N.$$

In this model, $X_1, X_2, \cdots$ are claim size random variables and N is the claim frequency random variable. The case of when N is a simple counting random variable with fixed parameters has been extensively studied. In this paper we generalize the above model to a mixed risk model where the random variable N depends on another random risk parameter. The parameter itself is assumed to be distributed over the population of risks under consideration in accordance with some distribution. There are several situations in which this might be a useful way to consider the distribution of N. For example, consider a population of insureds where various classes of insureds within the population generate numbers of claims according to different distributions.

2. Probability Distribution of S. Let $S = X_1 + \cdots + X_N$, where N is a counting random variable (r.v.) whose distribution depends on some random parameter Λ. In order to make the model mathematically attractive, we assume the following conditions.

1) $X_1, X_2, \cdots$ are identically distributed with common cumulative distribution function (c.d.f.) $F_X(x)$.

2) The random variables $N, X_1, X_2, \cdots$ are mutually independent (this implies that $\Lambda, X_1, X_2, \cdots$ are mutually independent).

We further denote the conditional probability density function (p.d.f.) of non-negative r.v. N given that $\Lambda = \lambda$ by $f_{N|\lambda}(n)$, for $n = 0, 1, 2, \cdots$. Also, we denote the c.d.f. of Λ by $F_\Lambda(\lambda)$. Using the law of total probability we can get the following