DIGITAL SOUP

Richard L. Francis

Southeast Missouri State University

1. Introduction. Word patterns often suggest similar patterns in the case for numbers. Among the most interesting of words are those which read the same from left to right as from right to left (e.g., eye, noon, level, refer, deified, etc.). These are generally identified as palindromes, a word stemming from the Greek "palindromos" which means "running back again." More precisely, they are instances of "reciprocal palindromes." In a digital setting, reciprocal palindromes give rise to such appealing forms as the reversible prime (e.g., 101 or 15551), the reversible square (e.g., 121 or 40804), and the reversible cube (e.g., 343 or 1331). But another variation on palindromes presents itself. Suppose a different but valid word is formed by reversing letters (such as "flow" and "wolf," "diaper" and "repaid," "reward" and "drawer," "reviled" and "deliver," etc.). These are ordinarily labeled "recurrent palindromes." They also suggest a variety of mathematical questions.

Accordingly, are there numbers in a given category which produce different numbers in the same category by the reversing of digits? Examples are rather quickly found and include the following:

1. counting numbers,
2. multiples of 3 or of 9, and
3. multiples of 11.

Note that a counting number is still a counting number when its digits are reversed, a multiple of 3 (or 9) remains a multiple of 3 (or 9) by the reversing (even the permuting) of digits, and that a multiple of 11 becomes a number of the same kind by reversibility. Many encounters with recurrent palindromic numbers are vastly more subtle. Some actually extend into the area of presently unsolved problems. A look at several familiar classes of numbers reinforces the concept.

2. Exact Squares. It is apparent that such pairs as 144 and 441 or 169 and 961 are squares having the desired reversibility feature. For convenience, denote the recurrent palindromic relationship (in a given set context) by the connective "r." Hence, 144 r 441