THE RATIONAL ZERO THEOREM EXTENDED

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Introduction. In college algebra classes the students are taught how to find zeros of polynomials such as $P(x) = x^4 - 4x^3 + 6x^2 - 4x + 5$. Normally the Rational Zero Theorem is applied to find the possible rational zeros to be ± 1 or ± 5. A quick check will reveal that none of these possibilities are zeros of $P(x)$. Since $P(x) = (x-1)^4 + 4 \geq 4$, $P(x)$ does not have real zeros. So the question is, how do we find the four complex zeros? A complex zero of the form $\alpha = a + bi$, where a and b are integers is usually given. The fact that the conjugate of α, denoted by $\overline{\alpha} = a - bi$ is also a zero is used to finish finding all the zeros.

Consider the fact that $5 = 2^2 + 1^2 = (2+i)(2-i)$. It would seem that $2+i$ and $2-i$ are divisors of 5. We can calculate $P(x) \div (x - (2+i))$ by synthetic division. We have

$2+i$	1	-4	6	-4	5
		$2+i$	-5	$2+i$	-5
	1	$-2+i$	1	$-2+i$	0

so that $2+i$ is a zero of $P(x)$. Therefore, $\overline{2+i} = 2-i$, the other divisor of 5, is a zero and we have

$2-i$	1	$-2+i$	1	$-2+i$
		$2-i$	0	$2-i$
	1	0	1	0

Finally, $P(x) = (x-2-i)(x-2+i)(x^2+1) = (x-2-i)(x-2+i)(x+i)(x-i)$.

The purpose of this paper is to generalize the Rational Zero Theorem to complex numbers of the form $\alpha = a + bi$, where a and b are integers. The set G of all numbers of this type are called Gaussian Integers. This set with the ordinary operations of addition and multiplication is an Integral Domain without order. We can define the notion of a divisor in the same way it is done for the set of integers. Denote the set of integers by I.

Some Number Theory in G.

Definition 1. If $\alpha = \beta\gamma$, $(\beta \neq 0)$ where α, β, $\gamma \in G$ then we say that β is a divisor (factor) of α and write $\beta | \alpha$.

Using this definition the divisibility properties of G are, by and large, the same as they are for the integers and are proven in the same manner. The notion of a prime in G is more general than the definition of a prime in I.