A-SETS AND ABCOHESIVE SPACES

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Definitions. A space M is *abcohesive at a point* p *with respect to a point* q if there exists an open connected set U such that p is a point in U and U is a subset of $M - \{q\}$. The space M is *abcohesive at a point* p if it is abcohesive at p with respect to q for each q in $M - \{p\}$. The space M is *abcohesive* if it is abcohesive at p for each p in M.

Remarks. If p is a non-cut point of M, and M is T_1 then M is abcohesive at each point q in $M - \{p\}$ with respect to p. Hence, if each point of M is a non-cut point of M, then M is abcohesive. Also, if M is a locally connected T_1 space, then M is abcohesive. Sierpinski space is locally connected but not abcohesive. However, Sierpinski space is not T_1. For the remainder of this paper, we will assume the space M is Hausdorff. If M is a continuum, then there exist two points p and q in M such that M is abcohesive at each x in $M - \{p\}$ with respect to p and at each x in $M - \{q\}$ with respect to q.

Theorem 1. The space M is abcohesive at each point q in $M - \{p\}$ with respect to p if and only if each component of $M - \{p\}$ is open.

Proof. Suppose M is abcohesive at each point q in $M - \{p\}$ with respect to p. Let C be a component of $M - \{p\}$, and let x be a point in C. Since M is abcohesive at x with respect to p, there exists an open connected set K such that $x \in K$ and $K \subset M - \{p\}$. But $K \subset C$, and hence C is open.

If the components of $M - \{p\}$ are open, then for each q in $M - \{p\}$, there exists a component C such that $q \in C$ and $C \subset M - \{p\}$. Therefore, M is abcohesive at q with respect to p.

Theorem 2. If M is an abcohesive connected space and C is a component of $M - \{p\}$, then p is a limit point of C.

Proof. Let C be a component of $M - \{p\}$. If p is not a limit point of C, then C is both open and closed in M. This involves a contradiction. Hence p is a limit point of C.

Theorem 3. If M is an abcohesive space, then the components of M are open.

Proof. If M is connected, then M is the only component of the space. If M is not connected, then let C be a component of M and let p be a point in $M - C$. By Theorem