SOME PROPERTIES RELATED TO DENDRITIC SPACES

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1. Introduction. L. E. Ward [1] investigated a large number of dendritic properties. He proceeded to establish sufficient conditions for these properties to be equivalent. Ward was apparently unable to show equivalence for all the properties he investigated in any space weaker than Peano continua. He also presented some other dendritic properties he did not investigate. We answer some of the questions he raised. Unless explicitly stated to the contrary, M will always denote a Hausdorff space.

2. Definitions. A *continuum* is a compact connected Hausdorff space. A connected space M is said to be *dendritic* if and only if each pair of distinct points of M is separated in M by a third point of M. A compact dendritic space is a *dendrite.* An element p of M is a *cutpoint* of M if $M - \{p\}$ is not connected. If $M - \{p\}$ is connected then p is a *non-cutpoint* of M. The space M is *paraseparable* if and only if M does not contain uncountably many disjoint open sets. A space M is said to be *strongly connected* if for each two points a and b in M, there exists a continuum L in M such that L contains a and b. p is an *end point* of the connected space M if each open set containing p contains an open set containing p whose boundary is degenerate.

The following theorem is known and can be found in Moore [2].

Theorem 1. If H is an uncountable set of cut points of the paraseparable connected set M, then some two points of H are separated in M by a third point of H.

Theorem 2. If M is a paraseparable strongly connected space and each continuum in M contains uncountable many cut points of M, then M is dendritic.

Proof. Let a and b be any two points of M. Since M is strongly connected, there exists a continum in M containing a and b. Let L be an irreducible continuum in M containing