ELEMENTARY APPLICATIONS OF COMPLEX NUMBERS

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In 1777, Leonard Euler introduced the imaginary number i with the property $i^2 = -1$. Perhaps the use of the word "imaginary" is infelicitous, however, it connotes the distrust with which complex numbers are viewed by beginning students. The purpose of this paper is to show that in many cases the evaluation of real integrals can be facilitated by using complex numbers and, as a bonus, sometimes two (real) results can be obtained from one computation.

The content of this paper is elementary but could be beneficial to some who teach lower level mathematics courses. Formal proof is not the point of the paper. Computations will be done formally and no justification, other than noting that the results are correct, will be given. For the reader who can manipulate complex numbers, this nonrigorous approach can be used as a source of motivation to study complex analysis – where both meaning and justification can be given. It is assumed that the complex number i can be manipulated like a real number.

The exponential function in complex analysis is defined for all complex numbers $z = x + iy$ by

$$e^z = e^x(\cos y + i \sin y) \ . \tag{1}$$

This definition can be motivated by formally substituting iy for y in the Maclaurin expansion

$$e^y = \sum_{n=0}^{\infty} \frac{y^n}{n!}$$

and rearranging the terms to get

$$e^{iy} = \sum_{n=0}^{\infty} \frac{(-1)^n y^{2n}}{(2n)!} + i \sum_{n=0}^{\infty} \frac{(-1)^n y^{2n+1}}{(2n+1)!} \ ,$$

$$= \cos y + i \sin y \ , \tag{2}$$