

**ON THE STRUCTURE OF THE HOPF REPRESENTATION
RING OF THE SYMMETRIC GROUPS**

Essam Abotteen

Emporia State University

The main purpose of this paper is to prove a structure theorem of the graded Hopf representation ring of the symmetric groups $R(S)$. We establish a Hopf ring isomorphism between $R(S)$ and the graded polynomial Hopf ring in an infinite number of variables

$$C = Z[y_1, y_2, \dots, y_k, \dots],$$

by using the λ -operations in $R(S)$ given in a previous paper [8] in terms of outer plethysms.

1. Introduction. In [8] λ -operations are introduced in the graded Hopf representation ring of the symmetric groups

$$R(S) = \{R(S_n) : n \geq 0\}$$

in terms of outer plethysms and it has been shown that with respect to these operations $R(S)$ is a special λ -ring. Zelevinsky [9] developed a complete structure theory of Hopf algebras satisfying the positivity and self adjointness, which is similar to the classical theory of Hopf algebras with commutative multiplication and comultiplication over a field of characteristic zero [7]. In this Hopf algebra approach, Zelevinsky showed $R(S)$ is isomorphic to the polynomial Hopf ring

$$C = Z[y_1, y_2, \dots, y_k, \dots]$$

in an infinite number of variables over the integers. Following an elegant proof of Liulevicius [5] we are going to reproduce the Zelevinsky structure theorem in the context of the λ -ring structure in $R(S)$.

Let S_n be the symmetric group of degree n . Let $R(S_n)$ denote the Grothendieck representation group of S_n , then we have a graded group

$$R(S) = \{R(S_n) : n \geq 0\}$$