MAXIMIZING THE SURFACE AREA OF AN N-DIMENSIONAL UNIT SPHERE

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Using the Dirichlet integral in n-dimensional Euclidean space, one can show that the volume of an n-dimensional sphere with radius r is given by

$$V_n(r) = \frac{\pi^{\frac{n}{2}} r^n}{\Gamma(\frac{n}{2}+1)} \ ,$$

where n is a positive integer. Of course, 'volume' V_1 is the length of the interval $[-r, r]$ and 'volume' V_2 is the area of the circle with radius r. So, $V_1(r) = 2r$ and $V_2(r) = \pi r^2$.

The surface area of an n-dimensional sphere with radius r is given by

$$\begin{aligned} S_n(r) &= V_n'(r) \\ &= \frac{n\pi^{\frac{n}{2}} r^{n-1}}{\frac{n}{2}\Gamma(\frac{n}{2})} \\ &= \frac{2\pi^{\frac{n}{2}} r^{n-1}}{\Gamma(\frac{n}{2})} \ . \end{aligned}$$

In [1], it was shown that $V_n(1)$ has a maximum value when $n = 5$. The purpose of this paper is to show that $S_n = S_n(1)$ attains a maximum value for $n = 7$. This will be accomplished by showing that

(a) $S_7 > S_6 > S_5 > S_4 > S_3 > S_2 > S_1$ and

(b) $\{S_n\}_{n=7}^{\infty}$ is a decreasing sequence.