A NOTE ON LOWER NEAR FRATTINI SUBGROUPS

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Theorem 1 in [1] reads, "Let H be a normal subgroup of a group G such that the order of H is prime. Let $\lambda(G)$ denote the set of all non-near generators of G. Then $\lambda(G) \cap H = \{1\}$ if and only if G nearly splits over H." The purpose of this note is to show that Theorem 1 in [1] may be improved as follows: If H is a normal subgroup of a group G and H is of prime order, more generally, if H is a finite cyclic normal subgroup of a group G, then $H \subseteq \lambda(G)$ and G does not nearly split over H. We also prove that if the condition "H is finite" is replaced by "H is infinite" in the above statement, then $\lambda(G) \cap H = \{1\}$ if and only if G nearly splits over H.

We first recall some definitions (see [1] or [2]).

Definition 1. An element g of a group G is a non-near generator of G if $S \subseteq G$ and $|G :< g, S > |$ is finite implies $|G :< S > |$ is finite. The set of all non-near generators of G, denoted by $\lambda(G)$, is called the lower near Frattini subgroup of G.

Definition 2. Let H be a normal subgroup of a group G. We say that G nearly splits over H if there exists a subgroup K of G such that $|G : K|$ is infinite, $|G : HK|$ is finite and

$$\bigcap_{g \in G} g^{-1}(H \cap K)g = \{1\}.$$

Lemma 1. If S is a subset of a group G and x is an element of G such that $|G :< S > |$ is infinite and $< x >$ is a finite normal subgroup of G, then $|G :< h, S > |$ is infinite for every $h \in < x >$.

Proof. Let $g \in G$ and $|x| = n$. Then

$$g < x, S >= g < x >< S >= \bigcup_{1 \leq i \leq n} gx^i < S > .$$

That is, any left coset of $< x, S >$ in G is a finite union of left cosets of $< S >$ in G and hence if $|G :< x, S > |$ is finite, then G is a finite union of left cosets of $< S >$ in G (i.e.