

THE ANALYTICAL MACHINERY OF SYMMETRY

Bella Wiener and Joseph Wiener

The University of Texas-Pan American

In 1951 one of the greatest mathematicians of this century, Hermann Weyl, delivered a series of remarkable lectures on symmetry at Princeton University. These lectures were designed for a very wide audience: faculty and students, mathematicians and nonmathematicians, people interested in natural sciences, and individuals interested in humanities. Weyl's captivating story, published in 1952 [1], contributed greatly to the understanding of the importance of the mathematical idea of symmetry for both mathematical and physical sciences. In 1967 another interesting book on symmetry appeared [2]. Its authors presented a beautiful and clear exposition of the theory of symmetric polynomials and their applications to such diverse algebraic topics as solving nonlinear equations and systems of equations, proving inequalities and identities, factoring complicated expressions, rationalizing the denominator, etc. Some observations on the use of symmetry in the study of functions have been made in [3]. This study was based on geometric methods and linear transformations. The purpose of the present paper is to survey some applications of symmetry to certain topics of algebra, analytic geometry, and calculus to exhibit the power of symmetry as an important analytical tool.

1. Solving Systems of Equations.

Consider first a Chicago All-Star Mathematics Problem: find all ordered pairs (x, y) for which

$$(1) \quad \begin{cases} x^2 + y^2 + x + y = 6 \\ xy + x + y = -1 . \end{cases}$$

This is an example of a symmetric system of equations. A system of two equations

$$P(x, y) = 0, \quad Q(x, y) = 0$$

is called symmetric if $P(x, y)$ and $Q(x, y)$ are symmetric polynomials in two variables, that is,

$$P(x, y) = P(y, x), \quad Q(x, y) = Q(y, x) .$$