SOLUTIONS

No problem is ever permanently closed. Any comments, new solutions, or new insights on old problems are always welcomed by the editor.

21. *Proposed by Stanley Rabinowitz, Westford, Massachusetts.*

Find distinct positive integers, a, b, c, d such that

$$a+b+c+d \quad + \quad abcd \quad = \quad ab+bc+ca+ad+bd+cd \quad + \quad abc+abd+acd+bcd\ .$$

Solution by the proposer.

My only solution is by computer search. The small solutions that I know of are:

$$a=2, b=4, c=16, d=70$$

and

$$a=2, b=4, c=22, d=32\ .$$

Comment by the editor.

This problem is still wide open!

22. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Without using Riemann sums prove that

$$\lim_{n\to\infty} n^{-3}\left(\sum_{k=1}^{n} k^{\frac{1}{2}}\right)^2 = \frac{4}{9}\ .$$

Solution I by the proposer.

From the Bernoulli Inequality we have

$$(1+k^{-1})^{\frac{3}{2}} > 1+\left(\frac{3}{2}\right)k^{-1}\ ,$$

$$(1-k^{-1})^{\frac{3}{2}} > 1-\left(\frac{3}{2}\right)k^{-1}\ .$$