

FIBONACCI DECIMAL NUMBER PATTERNS
VIA THE GENERATING FUNCTION

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The Fibonacci sequence is defined recursively by $F_1 = F_2 = 1$ and $F_{n+2} = F_{n+1} + F_n$ for $n \geq 1$. The first few terms of this sequence are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \dots . To six decimal places, the decimal expansion of $1/89$ is .011235. Similarly,

$$\frac{1}{9899} \simeq .0001010203050813213455, \quad \text{and}$$
$$\frac{1}{998999} \simeq .000001001002003005008013021034055.$$

Ignoring zeros, the occurrence of Fibonacci numbers in the above decimal expansions is apparent. If the decimal expansion of $1/89$ is carried out to one more digit, the digit in the seventh decimal place is 9, not 8. The purpose of this paper is to explain these phenomena.

In [1, p. 3], the generating function for the Fibonacci sequence