AN ANALYTICAL APPROACH TO A TRIGONOMETRIC INTEGRAL

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The integral of $\sin^2 x$ or $\cos^2 x$ between 0 and $\frac{\pi}{2}$ (or 0 and π) is usually calculated by changing the integrand according to the well-known half-angle formula. However, students with a phobia for trigonometry would prefer a simple geometric derivation of the answer suggested in [1]. The result is obtained immediately by noticing the equality of the areas of the regions under the graphs of $\sin^2 x$ and $\cos^2 x$ and by integrating the basic trigonometric identity $\sin^2 x + \cos^2 x = 1$. This idea is equivalent to the following analytical approach. Let

$$C = \int_0^{\frac{\pi}{2}} \cos^2 x \, dx, \quad S = \int_0^{\frac{\pi}{2}} \sin^2 x \, dx.$$

Since the substitution $x = \frac{\pi}{2} - u$ transforms either integral to the other, then $C = S$. Furthermore, integrating the basic identity between 0 and $\frac{\pi}{2}$ yields $C + S = \frac{\pi}{2}$, which implies $C = S = \frac{\pi}{4}$.