EXPANSIONS OF BAIRE SPACES

James Foran and Paul Liebnitz

University of Missouri-Kansas City

Let (X, T) be a topological space. Following Hewitt [3], if T' is a topology on X such that $T \subset T'$ we call (X, T') an expansion of (X, T). Several other authors [1, 2, 3, 4, 6, 7, 8] have subsequently studied the preservation of topological properties under expansions. In this note we consider the preservation of Baire spaces under expansions. A Baire space is a topological space in which every nonempty open set is of second category. Equivalently, a space is Baire if and only if none of its nonempty open sets is the union of countably many nowhere dense sets; this is true if and only if the intersection of every sequence of dense open sets is dense.

We show that an expansion of a Baire space need not be Baire, and that the supremum of a collection of Baire topologies need not be a Baire topology. We give some sufficient conditions under which an expansion of a Baire space is Baire. It would be interest-