NOETHERIAN INTEGRALLY CLOSED DUO RINGS ARE KRULL RINGS

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Introduction. A ring R with unity is called a duo ring if every right ideal of R is a left ideal and conversely. This is equivalent to the condition that $aR = Ra$ for all $a \in R$. When R is an integral domain, R has a (left and right) division ring D of quotients, and $xRx^{-1} = R$ for all nonzero $x \in D$. In this sense R is an invariant subring of D. In this note we characterize Noetherian, integrally closed duo domains [1] in terms of valuation rings exactly as commutative Krull domains are characterized. We continue in this manner to characterize duo domains which are unique factorization domains [3] as Krull domains in which the height 1 prime ideals are principal. We then show that when R has an Abelian group of divisibility, then R is integrally closed if and only if R is an intersection of valuation rings. Duo domains which are Prüfer domains with Abelian group of divisibility are characterized exactly as commutative Prüfer domains.

In what follows, R denotes a duo ring which is an integral domain, D its division ring of quotients. Prime ideals in duo rings have the same characterization as in the commutative case [10]. Thus P is a prime ideal of R if and only if $S = R \backslash P$ is a