Unitary Equivalence of Self-Adjoint Operators and Constants of Motion.

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In wave mechanics, the state of a dynamical system is represented at each instant of time by a wave function $\psi(q, t)$ which is the solution of

$$\frac{h}{2\pi i}\frac{\partial\psi}{\partial t} = -\boldsymbol{H}\psi\,. \tag{1}$$

When the Hamiltonian $\boldsymbol{H}$ does not contain t explicitly, we can write down the solution of (1), namely

$$\psi(q, t) = e^{-\frac{2\pi i}{h}\boldsymbol{H}t}\psi(q, 0)\,.$$

Since $e^{-\frac{2\pi i}{h}\boldsymbol{H}t}$ is a unitary operator, the function space representing the states at time t is obtained by operating the unitary operator $\boldsymbol{U}_t = e^{-\frac{2\pi i}{h}\boldsymbol{H}t}$ on the function space representing the states at time 0. Hence we may say that the motion of the dynamical system is represented by the unitary movement of the function space.

When the Hamiltonian $\boldsymbol{H}$ contains t explicitly, it is already proved that if $\{\psi_i(q, 0)\}$ is a complete normalised orthogonal system in the function space, then $\{\psi_i(q, t)\}$ is also a complete normalized orthogonal system in the function space, where $\psi_i(q, t)$ is the solution of (1) which is $\psi_i(q, 0)$ when $t = 0$.(1) Hence in this case also, we may consider the unitary movement of the function space.

Now the constants of motion are introduced as follows: An observable $\boldsymbol{A}$ is said to be a constant of motion when the elements a_{ij} of the matrix representing $\boldsymbol{A}$, is constant. When $\boldsymbol{H}$ does not contain t explicitly, $\boldsymbol{A}$ being independent of t, a_{ij} is defined by

(1) V. Fock, Zeitschrift für Physik, **49** (1928), 323.