On a new Definition of Vector and Parallel Displacement in Projective Differential Geometry.

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§ 1. Introduction.

In projective differential geometry as hitherto considered, as far as I know, a projective space was attached at every point in the base space, and the homogeneous coordinates were taken in each projective space. In this geometry the term "consecutive points" has no significance because of the homogeniality of the coordinates, nor, consequently, have the terms "differentiation," "infinitesimal quantity" etc. Besides, as to the relation between vectors and points in the projective space, we can find no necessary reason why this relation must be so considered.

In this paper, avoiding such considerations, we adopt the ordinary coordinates in each projective space instead of the homogeneous coordinates, and shall attempt to determine the relation between vectors and points in a rational manner.

For this purpose, we suppose that at every point in the base space, the projective space having a proper quadric is attached. And we give a new definition of summation of vectors such that the sum of the vectors is invariant for the projective transformations in the attached space. From this definition the relation between vectors and points will be determined as its necessary consequence. In this way we shall construct a system of projective differential geometry and find its various characteristics.

§ 2. The transformations of the coordinates in the base space and vector space, and those of the coefficients of quadric, in the projective space.

We consider an n-dimensional space X_n, in which we take the coordinates denoted by $x^1, \ldots, x^n$. At every point in this space X_n,