

# Wave Geometry.<sup>(1)</sup>

## (Geometry in Microscopic Space.)

By

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### § 1. Introduction.

According to Prof. Mimura,<sup>(2)</sup> we define the expression for the metric in the general microscopic space by

$$ds\psi = \gamma_i dx^i \psi \quad (1.1)$$

where  $\gamma$ 's are 4-4 matrices satisfying the equation

$$\gamma_i \gamma_j = g_{ij} I$$

and  $\psi$  is a 1-4 matrix given as a solution of the "unknown Dirac equation"—this equation will be obtained in § 5.

According to W. Pauli,<sup>(3)</sup> the most general expression for  $\gamma_i$  can be expressed as

$$\gamma_i = U h_i^\varepsilon \hat{\gamma}_\varepsilon U^{-1} \quad (1.2)$$

where  $U$  is any 4-4 matrix,  $h_i^\varepsilon$  can be any functions and  $\hat{\gamma}_\lambda$  the Dirac matrices.<sup>(4)</sup> Hereafter we assume that Latin suffices vary 1-4 and Greek suffices 1-5.

For the sake of brevity in the subsequent calculation and also bearing in mind certain future applications to projective relativity, we introduce  $\gamma_5$  such that

(1), (2) This Journal **5** (1935), 99.

(3) Ann. d. Phys. **18** (1933), 337.

(4)  $\hat{\gamma}_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \hat{\gamma}_2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \quad \hat{\gamma}_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$

$\hat{\gamma}_4 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}, \quad \hat{\gamma}_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$