Resolution of Identity.

By

Tôzirô OGASAWARA.

(Received April 20, 1935.)

Introduction.

Abstract Hilbert space $\mathfrak{H}$ is characterised by the following five properties:

(a) $\mathfrak{H}$ is a linear space.

(b) for any two elements $\mathfrak{f}$, $\mathfrak{g}$ of $\mathfrak{H}$ an inner product $(\mathfrak{f}, \mathfrak{g})$ is defined.

(c) $\mathfrak{H}$ is complete.

(d) $\mathfrak{H}$ is separable.

(e) for any positive integer n there exist n linearly independent elements.

As is well known, to every self-adjoint transformation H a resolution of identity $E(\lambda)$ corresponds, which is a projective transformation defined for $-\infty < \lambda < +\infty$ such that[1]

(1) $E(\lambda)E(\mu) = E(\mu)E(\lambda) = E(\lambda)$ for $\lambda < \mu$,

(2) $\lim_{\lambda \to \lambda_0 - 0} E(\lambda) = E(\lambda_0)$

(3) $\lim_{\lambda \to -\infty} E(\lambda) = O$, and $\lim_{\lambda \to +\infty} E(\lambda) = 1$.

And there exists a one-to-one correspondence between H and $E(\lambda)$ by the expression $(H\mathfrak{f}, \mathfrak{g}) = \int_{-\infty}^{+\infty} \lambda d\big(E(\lambda)\mathfrak{f}, \mathfrak{g}\big)$.

Using set U as parameter instead of λ, F. Maeda has generalised the concept of the resolution of identity as follows[2]: If $E(U)$ is a self-adjoint transformation which depends on Borel subset U of a Borel set V in a metric space and satisfies the following conditions, then $E(U)$ is said to be a resolution of identity.

---

(1) F. Riesz, Acta Szeged, **5** (1930), 23–54.

(2) F. Maeda, This journal, **4** (1934), 57. K. Friedrichs has considered $E(\Delta)$ which depends on an interval Δ instead of λ. See, Math. Ann., **110** (1934), 54–62.