An Extension of the Parallelism in X_n^{n-m} in X_n.

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§1. The parallelism in subspace has already been studied by many writers(1), but their methods have been based on the assumption that parallel vectors in X_n^{n-m} are obtained by a certain definite projection of the corresponding parallel vectors in X_n, accordingly the covariant derivatives in X_n^{n-m} are obtained by the projection of the corresponding covariant derivatives in X_n i.e. if we denote $\bar{v}'^{\lambda}$ is the projected vector of $\bar{v}^{\lambda}$ which is parallel (in X_n) to v^{λ},

$$\bar{v}'^{\lambda} = B^{\lambda}_{\mu}\bar{v}^{\mu} \quad \text{and} \quad \nabla'_{\omega} v^{\lambda} = B^{\mu\lambda}_{\omega\nu}\nabla_{\mu} v^{\nu} \quad \ldots\ldots\ldots\ldots \quad (1),$$

where
$$B^{\lambda}_{\mu} = \delta^{\lambda}_{\mu} - \sum_i \underset{i}{n}^{\lambda} \overset{i}{t}_{\mu}\ {}^{(2)} .$$

The parallelism in X_n^{n-m} so obtained does not become more extended than the original parallelism in X_n i.e. if the original parallelism is linear, affine, or Riemannian, the parallelism in X_n^{n-m} becomes at most the same.

Now the question arises: Is there any method by which the subspace X_n^{n-m} with a wider parallelism induced in X_n as we see in the case where any Riemannian space R_N can be induced in euclidean space $E_M\left(M \geqq \frac{N(N+1)}{2}\right)$? — for example, how can non-linear space, non-symmetric space or Weyl's space be induced in a Riemannian space?

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(2) In this paper we shall employ certain notations due to J.A. Schouten, *Der Ricci-Kalkül*, (1924).