Theory of Vector Valued Set Functions. II.

By

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In continuation of the work recorded in the preceding paper,(1) I investigated the unitary equivalence of the resolutions of identity. My theorems are analogous in content to those which Stone proved in his treatise.(2) But he treated the resolution of identity $E(\lambda)$ in connection with the corresponding self-adjoint transformation. Here I investigate the properties of the resolution of identity $E(U)$ from the standpoint of the vector valued set functions.

In part I, the variable U of the vector valued set function $\mathfrak{q}(U)$ is a Borel subset of a Borel set V in a metric space S which is half compact.(3) And I have assumed the uniform monotonity of the base $\sigma(U)$ of $\mathfrak{q}(U)$, in order that we may use the fundamental theorem between the integral and the derivative with respect to $\sigma(U)$.(4) But, O. Nikodym proved the fundamental theorem in the abstract space,(5) i. e. when $\phi(U)$ is absolutely continuous with respect to $\mu(U)$, there exists a point function $f(\lambda)$ which satisfies the following relation:

$$\phi(U) = \int_U f(\lambda) d\mu(U) .$$

Hence, if we say $f(\lambda)$ as the derivative of $\phi(U)$ with respect to $\mu(U)$, we have the fundamental theorem in the abstract space. Such a consideration is unsatisfactory from the standpoint of the theory of deri-

(1) F. Maeda, "Theory of Vector Valued Set Functions" this volume, 57-91. I shall refer to this paper as part I.

(2) M.H. Stone, *Linear Transformations in Hilbert Space*, (1932), 242.

(3) Cf. sec. 1, part I.

(4) Cf. footnote (3) of sec. 1, part I.

(5) O. Nikodym, Fund. Math., **15** (1930), 131-179.