On the Space which admits a given Continuous Transformation Group.

By

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We shall consider the space in which the parallelism of vectors is unaltered by all the transformations of a given continuous transformation group.

First, we will express the parallel displacement of vectors by Lie's symbols of the infinitesimal transformations, and from this we will obtain the necessary and sufficient conditions for the existence of a space which admits a given continuous transformation group. Further, if such a space exists, we will find all such spaces.

Next, we will see how any vector-and tensor-fields are transformed by the given continuous group, and obtain the relation between the parallel displacement and the transformation of the vector-field.

Lastly, in the case when the operators of the given transformation group are all unconnected, we will obtain the most general space which admits it.

I.

Let us consider a space $X_n^{(1)}$, and let the coordinates be $x^1, \ldots\ldots, x^n$, and the coefficients of connection be $\Gamma_{\mu\nu}^{\lambda}(x)$.

In this space let $v^{\lambda}(x)$ be a vector-field, and v'^{λ} be the vector at a point $P(x)$ which is parallel to the vector $v^{\lambda}(x+dx)$ at a point $Q(x+dx)$ in the neighbourhood of the point $P(x)$. Then we have

$$v^{\lambda}(x+dx)-v'^{\lambda} = -\Gamma_{\mu\nu}^{\lambda}v'^{\mu}dx^{\nu} ,$$

neglecting the terms higher than the 2nd. The left hand of this equation expresses the change of vector v'^{λ} when the vector v'^{λ} at the point

(1) In this paper we shall employ certain notations due to J. A. Schouten, *Der Ricci-Kalkül*, (1924).