On Kernels and Spectra of Bounded Linear Transformations.

By

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Let $\beta(E)$ be a completely additive, non-negative function of normal sets defined in a metric space R which is compact in itself, and be uniformly monotone almost everywhere (β)[1] in a β-normal set A. Let $\phi(E)$ be a complex valued set function which is absolutely continuous with respect to $\beta(E)$. When $\int_A |D_{\beta(E)}\phi(a)|^2 d\beta(E)$ is finite, then it is said that $\phi(E)$ belongs to the class $\mathfrak{L}_2(\beta)$. If we define the norm $||\phi||_\beta$ of $\phi(E)$ as

$$||\phi||_\beta = \left[\int_A |D_{\beta(E)}\phi(a)|^2 d\beta(E)\right]^{\frac{1}{2}},$$

and the inner product $(\phi, \psi)_\beta$ of two set functions $\phi(E)$ and $\psi(E)$ as

$$(\phi, \psi)_\beta = \int_A D_{\beta(E)}\phi(a)\overline{D_{\beta(E)}\psi(a)}d\beta(E),$$

then the space of all set functions of $\mathfrak{L}_2(\beta)$ is a Hilbert space.[2] I will denote this space also by $\mathfrak{L}_2(\beta)$.

In this paper, I first shew that all bounded linear transformations T defined in $\mathfrak{L}_2(\beta)$ are expressed in the integral form

$$T\phi(E) = \int_A D_{\beta(E')}\mathfrak{K}(E, a')D_{\beta(E')}\phi(a')d\beta(E'),$$

(1) That is, the β-value of the set of poles of $\beta(E)$ in A is zero. Cf. F. Maeda, this journal, **1** (1931), 3.

(2) Cf. my previous paper "On the Space of Real Set Functions," (this journal, **3** (1933), 1–42), where "linear manifolds" is equivalent to "closed linear manifolds" in this paper.