On the Space of real Set Functions.

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For a real point function $f(a)$ which belongs to the class $L_2(\beta)$, that is, $[f(a)]^2$ is integrable with respect to a set function $\beta(E)$ over a set A, the norm $||f||$ of $f(a)$ is defined as

$$||f|| = \left[\int_A [f(a)]^2 d\beta(E)\right]^{\frac{1}{2}},$$

and the inner product (f, g) of two point functions $f(a)$ and $g(a)$ of $L_2(\beta)$ is defined as

$$(f, g) = \int_A f(a)\, g(a)\, d\beta(E).$$

Then it is well-known that the set of all point functions of $L_2(\beta)$ is a Hilbert space, when two point functions, which differ at points of sets whose β-value is zero, are considered as identical.

Now let us say that a real set function $\phi(E)$, which is absolutely continuous with respect to $\beta(E)$, belongs to $L_2(\beta)$ when its general derivative $D_{\beta(E)}\phi(a)$ belongs to $L_2(\beta)$. By the fundamental theorem of the integration and differentiation, a unique general derivative $D_{\beta(E)}\phi(a)$ exists almost everywhere (β) and

$$\phi(E) = \int_E f(a)\, d\beta(E),$$

where $$f(a) = D_{\beta(E)}\phi(a).$$

Therefore, if we define the norm $||\phi||$ of $\phi(E)$ as

$$||\phi|| = ||f||,$$

and the inner product (ϕ, ψ) of $\phi(E)$ and $\psi(E)$ as

$$(\phi, \psi) = (f, g)$$

where $$g(a) = D_{\beta(E)}\psi(a),$$