Homogeneous Basis for Continuous Geometry.

By

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J. v. Neumann,(1) in his continuous geometry L, has defined the homogeneous basis, as a system of independent elements (a_i; $i=1,2,\ldots,m$) which are pairwise perspective and

$$a_1 \cup a_2 \cup \cdots \cup a_m = 1 . \tag{1}$$

When L satisfies the chain-condition, we have a homogeneous basis in which all the elements a_i are minimal. But when L does not satisfy the chain-condition, we cannot have such a homogeneous basis with *minimal* elements.

Thus we meet with a similar situation to that of ring-decomposition in algebra. A ring $\mathfrak{R}$, without radical, with minimum-condition for right ideals, is a direct sum of simple right ideals, i. e.

$$\mathfrak{R} = \mathfrak{a}_1 + \mathfrak{a}_2 + \cdots + \mathfrak{a}_n . \tag{2}$$

But when the ring does not satisfy the minimum-condition, we cannot decompose $\mathfrak{R}$ in a direct sum of *simple* right ideals as (2). To investigate the latter case, in a previous paper(2) I introduced a decomposition system of right ideals $\{\mathfrak{a}_U;\ U \in \{U\}\}$, where $\{U\}$ is a Boolean algebra. $\mathfrak{a}_U$ satisfies the following conditions:

(α) $$\mathfrak{a}_{U_1} \cap \mathfrak{a}_{U_2} = \mathfrak{a}_{U_1 \frown U_2} ;$$

(β) $$\mathfrak{a}_U = \mathfrak{a}_{U_1} + \mathfrak{a}_{U_2} + \cdots + \mathfrak{a}_{U_n} ,$$

when (U_1, $U_2, \ldots, U_n$) are independent and $U = U_1 \smile U_2 \smile \cdots \smile U_n$;

(γ) $$\mathfrak{a}_V = \mathfrak{R} ,$$

(1) J. v. Neumann [3], 30. The numbers in square brackets refer to the list given at the end of this paper.

(2) F. Maeda [1].