Generalized Geodesic Lines and Equation of Motion in Wave Geometry.

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§ 1. Introduction.

In Wave Geometry,[1] on reasonable grounds a four-component vector $u^i\,(=\psi^\dagger A\gamma^i\psi)$ has been taken as a particle momentum-density vector, and from u^i thus defined, the equation of motion of a particle is given by the following equation:

$$\frac{dx^1}{u^1}=\frac{dx^2}{u^2}=\frac{dx^3}{u^3}=\frac{dx^4}{u^4}\,.$$

On the other hand, in the general theory of relativity, the equation of motion of a particle is defined by the variational equation (geodesic lines):

$$\delta\int ds=0\,.$$

Thus there can be two lines of consideration for the definition of equation of motion of a particle. Here the problem arises: Is there any way to unify these two lines of consideration into one? In the present paper we shall inquire into the equation of motion of a particle from this point of view.

According to the principle of Wave Geometry, physical laws must be expressed by operators γ_i and the state function ψ. Therefore, when we take a variational equation as the equation of motion, it is natural to think that the geodesic line (the equation of motion of a free particle in a field) must be given by an expression depending on both γ_i and ψ instead of $\delta\int ds=0$, which depends only on g_{ij}.

By obtaining this generalized geodesic line, we shall show that *the physical laws in the macroscopic world hitherto obtained in Wave-Geometry (Cosmolog, etc.) and the law unifying gravitation and electromagnetism (of the Born type) are unified in the same principle.*

(1) T. Iwatsuki and T. Sibata: This Journal, **10** (1940), 247 (W. G. No. 41).
T. Iwatsuki, Y. Mimura and T. Sibata: Ibid., **8** (1938), 187 (W. G. No. 27).
K. Sakuma: Ibid., **11** (1941), 15 (W. G. No. 42), and other papers entitled "Cosmology in Terms of Wave Geometry."