Almost Periodic Functions in Groups.

By

Tôzirô OGASAWARA.

(Received June 30, 1941.)

The object of this paper is to study the theory(1) of J. v. Neumann's almost periodic functions (a. p. f.), with the view that the theory is reduced to the study of the space of continuous functions (a. p. f.) on a uniquely determined bicompact topological group. This idea originates from A. Weil(2) and E. R. van Kampen.(3) In Part 1 we study the writings of F. Wecken(4) and S. Bochner(5) on a. p. f. In Part 2 we study the theory(6) of Bochner-Neumann's vector-valued a. p. f. under the same idea.

Here I express my hearty thanks to Prof. F. Maeda for his kind guidance.

Part 1.

§ **1.** Here we are concerned with J. v. Neumann's a. p. f. in an abstract group G, the element of which we denote by a, b, x, y, z. Let T be any set of a. p. f., and let T^* be the smallest group-invariant set of a. p. f. containing T, i. e. the set of all $f(axb)$, $f\in T$. Let H be the set of elements a of G such that $f(a)=f(e)$ for all $f\in T^*$, where e denotes the unit element of G; then H is the invariant subgroup of G. We assume $H=(e)$, for the general case is reduced to this. We make G a topological group by introducing neighbourhoods

$$U_{(f_1,f_2,\dots,f_n;\,\epsilon)}(a)=\big(x\,;\ |f_i(x)-f_i(a)|<\epsilon,\ f_i\in T^*,\ i=1,2,\dots.n\big)\,.$$

Let $\rho_f(a,b)$ be the translation function of a. p. f. $f(x)\in T$; that is,

$$\rho_f(a,b)=\operatorname*{l.\,u.\,b.}_{x,y\in G}|f(xay)-f(xby)|\,.$$

Put

$$V_{(f_1,f_2,\dots,f_n;\,\epsilon)}(a)=\big(x\,;\ \rho_{f_i}(a,x)<\epsilon,\ f_i\in T,\ i=1,2,\dots.n\big)\,;$$

then, by means of these neighbourhoods, G becomes another topological space.

---

(1) J. v. Neumann, Trans. Amer. Math. Soc. **36** (1934), 445–492.
S. Bochner and J. v. Neumann, Trans. Amer. Math. Soc. **37** (1937), 21–50.
E. R. van Kampen, Annals of Math. **37** (1935), 78–91.

(2) A. Weil. C. R. Paris, **200** (1935), 38–40.

(3) loc. cit.

(4) F. Wecken, Math. Zeitschrift **45** (1939), 377–404.

(5) S. Bochner, Annals of Math. **40** (1939), 769–799.

(6) loc. cit.